INVESTIGATING THE MAXIMUM DETERMINANT FOR AN NXN MATRIX

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Abstract

It is known that for any $n \times n$ matrix $A_n = (a_{ij})$; $|a_{ij}| \leq m$, $m \in \mathbb{Z}^+$, $|detA_n| \leq m^n n^{n/2}$, (Garling, 2007). Therefore, $m^n n^{n/2}$ is an upper bound of determinants of all matrices A_n which satisfy the above conditions.

In this research, we determine the maximum determinant of an $n \times n$ matrix A_n, where $A_n = (a_{ij})$, $a_{ij} \in \{0,1,2,3\}$, for n =1, 2, 3, 4 and 5 using the determinant function formula and expansion using minors. For an $n \times n$ $\{0,1,2,3\}$–matrix, the maximum determinants for $n = 1,2,3,4,5$ were found to be $3,9,54,243$ and 972 respectively. The number of distinct $\{0,1,2,3\}$–matrices attaining the maximum determinant for $n = 1,2,3,4,5$ are $1,14,6,24$ and 120 respectively. For an $n \times n$ matrix $A_n = (a_{ij})$; $a_{ij} \leq m$, $n, m \in \mathbb{Z}^+$, $|det\, A_n| \leq (n-1)m^n$ with equality if and only if A_n has one and only one zero entry in each row and one and only one zero entry in each column, all the other entries in this matrix are equal to m. The number of distinct such matrices attaining the maximum determinant is $n!$, $n > 2$.

Key words: $\mathrm{n \times n}$ – matrix, maximum determinant, supremum determinant, minor matrix.